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Analysis of steady forced convection in thermally developing turbulent duct flows

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Nomenclature

D	=	internal duct diameter, [= $2r_i$], m
f	=	friction factor, defined by Equation (13)
$\bar{f}_k$	=	eigen coefficient, defined by Equation (5.b)
h	=	internal fluid-wall heat transfer coefficient, $W/m^2\ K$
k	=	thermal conductivity, $W/m\ K$
N	=	number of terms defining a series
N_k	=	norm of eigenvalue problem, defined by Equation (4.c)
$Nu(Z)$	=	local Nusselt number, defined by Equation (11)
Nu_∞	=	thermally developed Nusselt number, defined by Equation (12)
Pr	=	Prandtl number, [= ν/α]
Pr_t	=	turbulent Prandtl number, [= ϵ_m/ϵ_h]
R	=	dimensionless normal co-ordinate, [= r/r_i or $1 - y^+/R^+$]
R_e	=	Reynolds number, [= $\bar{u}D/\nu$]
R^+	=	dimensionless factor, [= $0.5Re\sqrt{f/8}$]
r	=	radial distance, m
ri	=	duct internal radius, m
$T(r,z)$	=	temperature within the fluid, K
T_i	=	reference fluid inlet temperature, K
T_w	=	constant wall surface temperature,- K
T_∞	=	ambient temperature, K
$U(R)$	=	dimensionless velocity profile, defined by Equation (2.a)
$u(r)$	=	velocity profile across the duct, m/s
$\bar{u}$	=	mean velocity of fluid, m/s
$u+$	=	dimensionless turbulent velocity profile, defined by Equation (2.a)
$W(R)$	=	dimensionless coefficient, defined by Equation (2.b)
y^+	=	dimensionless distance from the wall, [= $0.5(1 - R)Re\sqrt{f/8}$]
z	=	axial distance, m
Z	=	dimensionless axial distance, [= $(D/r_i)^2\alpha z/(\bar{u}D^2)$, or $(z/D)/Pe$]

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Greek symbols

α	=	fluid thermal diffusivity, m^2/s
$\in$	=	absolute modulus of variation, %
$\in(R)$	=	turbulent eddy viscosity, defined by Equation (2.a), m^2/s
ν	=	kinematic viscosity, m^2/s
$\Theta(R,Z)$	=	dimensionless fluid temperature,- defined by Equations (2.a) and (7)
$\Theta_b(Z)$	=	dimensionless fluid bulk temperature, defined by Equation (8)
μk	=	kth eigenvalue of Equation (3.a)
Ψ_k	=	eigenfunction of kth eigenvalue, defined by Equation (3)

Subscripts

b	=	bulk
f	=	fluid
h	=	heat
I	=	inlet
i,j,k	=	order of eigenquantities
m	=	momentum
t	=	turbulent
ts	=	theoretical solution
w	=	wall
∞	=	remote regions, i.e. infinity

Note: The symbols defined above are subject to alteration on occasion

Introduction

Studies of steady forced convective heat transfer inside circular ducts with fully developed turbulent flow and constant wall surface temperature are of immense technological importance, as they occur frequently under normal operating conditions in a wide variety of heating and cooling devices. In such cases, when compared with applications involving the constant wall heat flux boundary condition, improved systems performance and ease of control are usually reported. However, although the inherent advantages of turbulent forced convective heat transfer are being exploited, there is no fundamental theory available in the open literature to determine turbulent velocity distributions from a purely theoretical approach. In general, empirical and/or semi-empirical models are relied on to correlate turbulent velocity distributions inside ducts and channels. In circular ducts, flow and heat transfer characteristics for the turbulent regimes are being explored in great detail, as this geometry finds widespread use in diverse applications, and for practical purposes, both flow and heat transfer correlations that are derived for use with circular ducts are readily applicable with reasonable accuracy to other geometries ducts, provided the hydraulic diameter of the latter is appropriately substituted for the diameter of the former.

Recently, numerous researchers have reported findings detailing information on both analytical[1-5] and experimental[5-7] studies of steady and unsteady forced flows inside circular ducts and rectangular channels, within the laminar and turbulent flow regimes under different types of boundary condition. In some of the earlier works, the initial transients associated with unsteady flows are neglected and quasi-steady approaches applied to approximate the thermal response of the system under periodic disturbance, while in others, simplified solutions were evaluated based on the assumption of slug flow velocity profiles. Other attempts[5] have led to the solution of complex eigenvalue problems that are not of the conventional Sturm-Liouville type, with the main tasks being formulating solution techniques for evaluating the complex eigen quantities, particularly in cases where time-dependent functions are not always eliminated from the initial transient definitions.

In contrast to the foregone studies, this work provides new information on an innovative analytical approach, utilizing ideas associated with the generalized integral transform technique[5,8-10] to obtain benchmark results for quantities of practical interest in the accurate design of heat transfer devices employing a circular duct geometry. The use and practical applicability in the analysis of typical engineering problems is demonstrated by treating the case of hydrodynamically developed steady forced convection inside a circular duct with turbulent flow and constant wall surface temperature. In such an application, the associated auxiliary eigenvalue problem for the thermal response of the fluid will be analytically handled through the advanced sign-count method[11] providing eigenquantities with a user-prescribed accuracy. For the case here advanced, validation of the formulated approach will be sought through critically comparing the analytically derived thermally developed Nusselt numbers with those obtained from reliable empirical correlations[12-14] using the latter as the basis of comparison. In such analyses, the maximum in absolute differences are selected as accuracy indicators for predication, since such norms are rather strict when compared to the norms of the root-mean-squared residuals, thereby guaranteeing benchmark predicated results.

Practical applications of this work include, among others, an analytical tool for the prediction of local and thermally developed heat transfer parameters, as well as bulk, radial and axial temperature distributions within the fluid as it progresses along the flow passage – which are critical parameters for the accurate design of thermal energy conversion devices. In addition to circular ducts, the analyses are readily applicable to other geometries and flow regimes by appropriate transformations. The information presented here, based on their benchmark nature, can serve as a fundamental source for comparing results of new simulation techniques in heat transfer analysis.

Theoretical analysis

Steady forced convection with fully developed turbulent flow and constant surface wall temperature is considered inside the thermal entry region of a circular duct, wherein the turbulent velocity profile is established by a three-layer distribution[15] and the eddy diffusivity by a two-layer algebraic empirical model[16]. Along the thermal entrance region, the thermal response of the flowing fluid is evaluated for the case where axial conduction, free convection and viscous dissipation effects are neglected. Therefore, constant fluid thermophysical properties, the energy equation relative to that region can be written in dimensionless form as:

$$W(R)\left[\frac{\partial \Theta(R,Z)}{\partial Z}\right] = \frac{\partial}{\partial R}\left[\epsilon(R)\, R\left(\frac{\partial \Theta(R,Z)}{\partial R}\right)\right],\quad 0 < R < 1,\ Z > 0 \qquad (1.a)$$

with boundary conditions given as:

$$\frac{\partial \Theta(0,Z)}{\partial R} = 0 \qquad (1.b)$$

and

$$\Theta(1,Z) = 0 \qquad (1.c)$$

and inlet condition as:

$$\Theta(R,0) = 1 \tag{1.d}$$

while the dimensionless groups are defined as:

$$\Theta(R,Z) = \frac{T(r,z) - T_w}{T_i - T_w}, \quad R = \frac{r}{r_i}, \quad U(R) = \frac{u(r)}{\bar{u}} = u^+ \sqrt{f/8} \tag{2.a}$$

$$W(R) = R\,U(R), \quad Z = \frac{(D/r_i)^2\,\alpha\,z}{\bar{u}\,D^2}, \quad \epsilon(R) = 1 + \frac{\epsilon_h}{\alpha} = 1 + \frac{Pr}{Pr_t}\left(\frac{\epsilon_m}{\nu}\right) \tag{2.b}$$

with ϵ_m being eddy diffusivity of momentum[16] and u^+ the three-layer turbulent velocity distribution[15].

The integral transform method is a "well-known" classical approach in the analytical solution of certain classes of linear transformable diffusion problems[17-19]. The treatise, Mikhailov and Özişi[11] compiles most of the available work in the exact analysis of heat and mass diffusion, following the ideas of integral transformation. During the last two decades, after the work of Özişi and Murray[19], this approach was progressively extended to allow for approximate analytical solutions of a much wider range of a priori non-transformable problems, as reviewed in different sources[8,20]. More recently, however, this approach gained a hybrid analytical-numerical structure, offering user control accuracy and quite efficient computational performance for a wide variety of problems, some of which are classified and systematically arranged with several applications[8], including the non-linear formulation of interest in heat and fluid flow applications.

In the present work, manipulation of ideas associated with the generalized integral transform technique allows the selection of an auxiliary eigenvalue problem – which is a special case of the classical Sturm-Liouville system, as:

$$\frac{d}{dR}\left[\epsilon(R)\,R\,\frac{d\psi_k(R)}{dR}\right] + \mu_k^2\,W(R)\,\psi_k(R) = 0 \tag{3.a}$$

with boundary conditions:

$$\frac{d\psi_k(0)}{dR} = 0 \tag{3.b}$$

and

$$\psi_k(1) = 0 \tag{3.c}$$

wherein solutions of the related eigenvalues, *µk's*, and eigenfunctions, *yk's*(*R*), are at this point assumed known, through application of the advanced sign-count method[11], providing safe, guaranteed and automatic computation of as many eigenquantities as desired with a user-prescribed accuracy.

The auxiliary eigenvalue problem (i.e., System (3)), allows definition of an integral transform pair for the function $\Theta(R,Z)$ as:

$$\Theta(R,Z) = \sum_{k=1}^{\infty} \frac{\psi_k(R)}{N_k^{1/2}} \bar{\Theta}_k(Z) \tag{4.a}$$

and

$$\bar{\Theta}_k(Z) = \frac{1}{N_k^{1/2}} \int_0^1 W(R)\, \psi_k(R)\, \Theta(R,Z)\, dR \tag{4.b}$$

and with the use of a symmetric kernel, the normalization integral can be stated as:

$$N_k = \int_0^1 W(R)\, \psi_k^2(R)\, dR \tag{4.c}$$

The main problem, (Equation 1.a), can now be transformed, after appropriate algebraic manipulations, involving use of eigenvalue problem, (Equation 3.a), and all boundary and initial conditions to yield:

$$\frac{d\bar{\Theta}_k(Z)}{dZ} + \mu_k^2\, \bar{\Theta}_k(Z) = 0 \tag{5.a}$$

and

$$\bar{\Theta}_k(0) = \bar{f}_k = \frac{1}{N_k^{1/2}} \int_0^1 W(R)\, \psi_k(R)\, dR \tag{5.b}$$

System (5) now has an explicit analytic solution given by:

$$\bar{\Theta}_k(Z) = \bar{f}_k\, e^{-\left(\mu_k^2 Z\right)} \tag{6}$$

and when operated on by the inversion formula, (Equation 4.b), yields the following expression for the thermal response of the fluid within the temperature field:

$$\Theta(R,Z) = \sum_{k=1}^{\infty} \frac{\bar{f}_k}{N_k^{1/2}}\, \psi_k(R)\, e^{-\left(\mu_k^2 Z\right)} \tag{7}$$

In the work here advanced, other parameters/quantities of practical relevance are evaluation of the dimensionless bulk fluid temperature, $\Theta_b(Z)$ and the local Nusselt number, $Nu(Z)$, as they vary along the thermal entrance region of the duct under the prescribed flow and boundary conditions. Based on their universal definitions, the local bulk fluid temperature and Nusselt number, at any measurement station within the thermal entrance region can, respectively, be evaluated as:

$$\Theta_b(Z) = \left[\frac{\int_0^1 W(R)\ \Theta(R,Z)\ dR}{\int_0^1 W(R)\ dR} \right] = 2 \sum_{k=1}^{\infty} \bar{f}_k^2\ e^{-\mu_k^2 Z} \tag{8}$$

and

$$Nu(Z) = \left[\frac{h(Z)\ D_h}{k_f} \right] = \left[\frac{-2 \left[\frac{\partial \Theta(1,Z)}{\partial R} \right]}{\Theta_b(Z)} \right] \tag{9}$$

From (Equation 7), at the internal fluid-to-solid interface, (i.e. at the wall):

$$\left. \frac{\partial \Theta(R,Z)}{\partial R} \right|_{R=1} = \left. \sum_{k=1}^{\infty} \frac{\bar{f}_k}{N_k^{1/2}}\ e^{-(\mu_k^2 Z)}\ \frac{d\psi_k(R)}{dR} \right|_{R=1} \tag{10.a}$$

or

$$\left(\frac{\partial \Theta(R,Z)}{\partial R} \right)_{R=1} = -\sum_{k=1}^{\infty} \bar{f}_k^2\ \mu_k^2\ e^{-\mu_k^2 Z} \tag{10.b}$$

and after introducing Equations (8) and (10.b) into Equation (9), it can be shown that[5]:

$$Nu(Z) = \left[\frac{\sum_{k=1}^{\infty} \bar{f}_k^2\ \mu_k^2\ e^{-\mu_k^2 Z}}{\sum_{k=1}^{\infty} \bar{f}_k^2\ e^{-\mu_k^2 Z}} \right] \tag{11}$$

Thermally developed Nusselt numbers (theoretically)

Cases of steady fully developed turbulent flow in a circular duct with constant wall surface temperature occur frequently, and hence it is very important to evaluate limiting heat transfer coefficients and, in addition, to be fully aware of the direct implications of constant wall surface temperature, as opposed to a constant wall heat-flux boundary condition. In the past, an analysis for a circular duct was executed for the constant wall surface temperature by Sleicher and Tribus[21]. The momentum and heat transfer characteristics for transitional and turbulent flows in the entrance region were predicted by extending the laminar flow approach – through the adoption of an eddy-viscosity and turbulent Prandtl number concept. The resulting eigenvalues and eigenfunctions were numerically determined and applied in the evaluation of entrance region effects, as was the approach for laminar flow situations. However, it was later shown[22] that such a procedure was not generally accurate, although widely used, since the correction factor was proven to be multi-variable dependent, with both the Reynolds and Prandtl numbers having independent, but marked, influences.

In the novel theoretical approach advanced in this presentation, the thermally developed Nusselt numbers, Nu_∞, are evaluated as a function of the temperature field, thereby being totally dependent on the computed eigenquantities defining the thermal characteristics of the flowing fluid within that region of the duct. At remote downstream positions far removed from the very inlet, (i.e. as $Z \rightarrow \infty$), and referring to the local Nusselt number expression of (Equation 11), it can be shown after a certain degree of algebraic manipulation that:

$$Nu(Z)_{Z\rightarrow\infty} = Nu_\infty = \mu_1^2 \qquad (12)$$

which is explained as follows. On close inspection of (Equation 11), it can be inferred that for $k \geq 2$, the quantity ($\mu_k^2 = \mu_{k-1}^2$), will never be negative, and is therefore representative of a continuously increasing positive value, since succeeding eigenvalues are always greater that preceding eigenvalues (i.e. $\mu_k > \mu_{k-1}$), and at large values of Z, (Equation 11) reduces to (Equation 12)[5]. Therefore, unlike other approaches cited in the literature – with their respective limitations, it has be been shown by the novel approach here advanced that: in steady forced convection with turbulent flow and constant surface wall temperature, the limiting or thermally developed Nusselt numbers are obtainable as the magnitude of the first eigenvalues squared – and are Reynolds and Prandtl number dependent. A detailed description of the formulation on the novel approach is provided in the work of Brown[5].

Empirical correlations

In order to complement the theoretical work here advanced, benchmark analytical values of thermally developed Nusselt numbers had to be compared with reliable experimental and/or empirical data[12-14], under similar flow conditions, thereby verifying the benchmark nature associated with the theoretical solutions. To accomplish such tasks, complementary analyses were carried out with the use of proven reliable empirical correlations, wherein a single multivariable empirical expression was employed for evaluating the frictional effects as the fluid progresses through the thermal entry region. With this approach, the residuals introduced by the multivariable friction factor expression were transferred across the complete solution domain entertained by the verification analyses, and as such, had zero influence on both the predication accuracy of the novel technique and the empirical values.

Friction factor

In the definition of frictional effects associated with the flow phenomena within the thermal entrance region of the circular duct, the empirical correlation suggested by Filonenko[23] was employed throughout the analyses (both theoretical and empirical). For steady, fully developed turbulent flow inside smooth circular ducts with constant surface wall temperature, this correlation was shown by Bhatii and Shah[3] to have exhibited excellent accord with the combined Prandtl-Karman-Nikuradse (PKN) correlation, whenever its applied within the suggested applicable Reynolds number range. Thus, in the present work, the friction factor is stated as[23]:

$$f = [1.58 \ln Re - 3.28]^{-2}, \quad \text{for} \quad 10^4 \le Re \le 10^7 \tag{13}$$

and for the indicated Reynolds number range, it was shown[3] to produce results with uncertainties within ± 1.8 per cent of the Prandtl-Karman-Nikuradse expression.

Thermally developed Nusselt numbers (empirically)

It is well known that a large number of correlations, both theoretical and empirical, are available in the open literature[3] for evaluating the thermally developed Nusselt numbers for cases of forced convection inside circular ducts with turbulent flow and constant wall surface conditions. However, on the basis of proven applicability range coupled with accuracy and reliability of predictions, the combined Prandtl[12] Taylor[13] and Gnielinski[14] correlations are selected for comparing their results with those from the novel theoretical approach. The correlations are, respectively, expressed as follows; Prandtl[12] Taylor[13] and Gnielinski[14]:

$$Nu_\infty = \left[\frac{(f/2)\, Re\, Pr}{1.0 + 5.0\,(f/2)^{1/2}\,(Pr - 1)} \right],$$

$$\textit{for} \quad Pr \le 10, \quad \textit{and} \quad 5 \times 10^3 \le Re \le 5 \times 10^6 \tag{14}$$

where (Equation 14) is based on a two-layer model (laminar sub-layer and turbulent core), with uncertainties of +14.9 per cent and –11.1 per cent, respectively, of the Gnielinski[14] correlation. The fully developed Nusselt number correlation given by Gnielinski[14] is:

$$Nu_\infty = \left[\frac{(f/2)\,(Re - 1000)\, Pr}{1 + 12.7\,(f/2)^{1/2}\,(Pr^{2/3} - 1)} \right], \tag{15.a}$$

$$\textit{for} \quad 0.5 \le Pr \le 2000, \quad \textit{and} \quad 2300 \le Re \le 5 \times 10^6$$

and

$$\mathrm{Nu}_\infty = \left[0.0214 \left(\mathrm{Re}^{0.8} - 100\right) \mathrm{Pr}^{0.4} \right], \quad (15.b)$$

$$for \quad 0.5 \le Pr \le 1.5 \,, \quad and \quad 10^4 \le Re \le 5 \times 10^6$$

where the first Gnielinski expression, (Equation 15.a), has been generally selected as the common basis of comparison for most other correlations, and the second, (Equation 15.b) shown to agree with the first, (Equation 15.a) within +4 per cent and –6 per cent accuracy, respectively[3].

Results and discussions

A novel approach, based on an extension of the generalized integral transform technique has been formulated for obtaining hybrid analytic solutions of fully developed steady forced convection inside the thermal entrance region of circular ducts with turbulent flow and constant wall surface temperature conditions. With this model, the auxiliary eigenvalue problem associated with the thermal response of the fluid within the temperature field can be resolved analytically with a user prescribed accuracy, providing benchmark results for parameters of practical relevance in the accurate design and simulation of critical heat transfer devices. Further, on the merit of its hybrid nature, computational tasks were significantly reduced compared to other methods, requiring 80 to 100 terms for convergence, in most cases, while in regions far removed from the very inlet only few terms are required when heat transfer parameters were being evaluated to an accuracy of five decimal places – noting that the analyses are bounded by a global error control scheme in addition to the convergence criteria of 10^{-5}.

In assessing the accuracy associated with predications from the novel approach, its hybrid analytic solutions of thermally developed Nusselt numbers are compared with results from three different empirical correlations over a wide range of Reynolds numbers, as shown in Table I. An overall inspection of Table I shows satisfactory agreement prevailing between analytically and empirically evaluated thermally developed Nusselt numbers over the entire Reynolds number range. In most cases, deviations are shown to be within 7.0 per cent, with maximum differences of 8.73 per cent and 9.58 per cent respectively, appearing for the Gnielinski's and Prandtl-Taylor's correlations at the lowest Reynolds number. As noted, however, in all the cited cases, the difference diminishes with increasing Reynolds number and at the largest Reynolds number, the predicated value of the novel approach and that of the Gnielinski's correlation are equivalent, while a difference of 6.93 per cent exists between the analytic and the Prandtl-Taylor correlation. In addition, marked variations are shown to be associated with low turbulent Reynolds numbers, since the values are outside the suggested range of the turbulence model. In general, however, the cited satisfactory agreements provide authenticity to the analytical approach in accurately predicting the thermal response of the fluid within the temperature field for steady turbulent flow in circular ducts with constant surface wall temperature.

In Figures 1 and 2 the theoretical solutions for the variations in local bulk fluid temperature are illustrated as a function of the dimensionless streamwise coordinate within the thermal entry region for a selected Reynolds number range, covering low, intermediate and high turbulent flow regimes. As shown, the local fluid bulk temperature takes on its maximum value at the inlet and then decays exponentially with increasing distance along the duct. In each case,

Re	${}^{ts}Nu_{\infty}$	${}^{14}Nu_{\infty}$	${}^{12,13}Nu_{\infty}$	$\in^{14-ts}$	$\in^{12,13-ts}$
6,000	21.53	19.65	19.18	8.73	9.58
7,000	23.94	22.41	21.65	6.39	6.83
8,000	26.29	25.05	24.04	4.72	4.95
9,000	28.57	27.58	26.38	3.47	3.59
10,000	30.80	30.03	28.66	2.50	2.56
20,000	51.27	51.77	49.42	0.98	0.97
50,000	103.48	105.08	101.55	1.55	1.52
100,000	178.73	180.24	175.10	0.84	0.84
200,000	311.81	311.43	301.92	0.12	0.12
500,000	658.88	649.78	620.00	1.38	1.40
1,000,000	1,143.85	1,143.85	1,069.74	0.00	6.93

Table I.
Nusselt numbers for thermally developed steady forced convection in circular ducts for constant wall surface temperature and turbulent flow with $Pr = 0.71$, $Pr_t = 0.86$

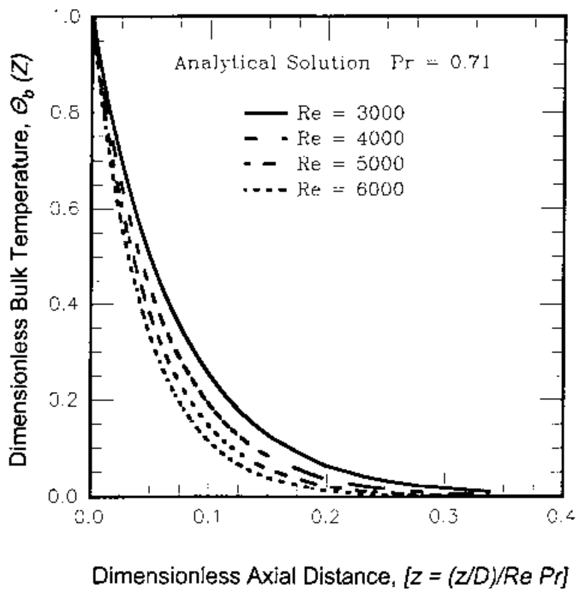

Figure 1.
Fluid bulk temperature variation in the thermal entry region of a circular duct for constant wall surface temperature and turbulent flow with $Pr = 0.71$ and $3,000 \le Re \le 6,000$

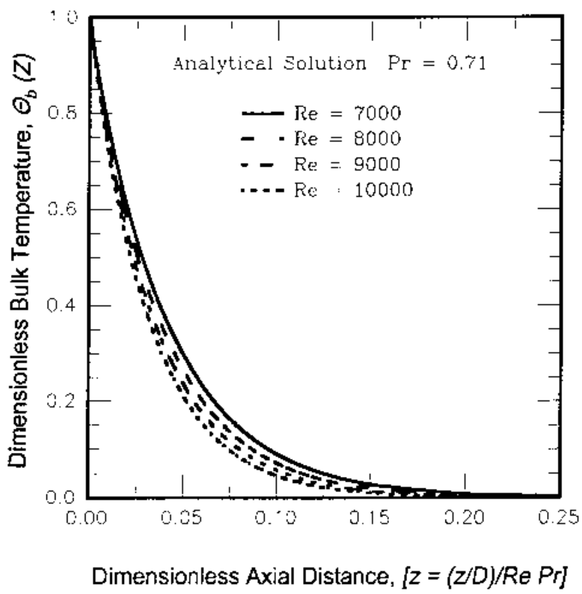

Figure 2.
Fluid bulk temperature variation in the thermal entry region of a circular duct for constant wall surface temperature and turbulent flow with $Pr = 0.71$ and $7{,}000 \leq Re \leq 10{,}000$

the decaying rate of the fluid bulk temperature appears to be strongly influenced by the Reynolds number; exhibiting increased slopes at larger Reynolds numbers. In general, the fluid bulk temperature experiences sharp decrease in the immediate neighbourhood of the inlet, and then tends to be gradual until minimum values are attained at downstream locations far removed from the inlet.

Figures 3 and 4 demonstrate the analytical results for the local Nusselt number variations corresponding to the flow conditions illustrated in Figures 1 and 2. As a function of the dimensionless streamwise co-ordinate Z, the local Nusselt number attains a maximum value at the inlet and then decreases monotonically with increasing downstream distance. The rate of decrease is observed to be extremely sharp in the immediate neighbourhood of the inlet, but becomes increasingly more gradual as the curves tend to level-off and the local Nusselt number approaches the thermally developed value, Nu_∞ at an extended downstream position, following which the value is invariant with position.

In Figures 3 and 4, the space separating adjacent pairs of curves appear to be independent of axial position, except for regions in the immediate neighbourhood of the inlet. Therefore, if a functional relationship is chosen as $g_1(Z/D)$ for the invariant regions, it can be inferred that $g_1(Z/D)$ is a constant. In such cases, if the parallelism associated with the illustrative curves (i.e. Figures 3 and 4) can generally be thought of as being indicative of the functional relationship $Nu(Z) = g_1(Z/D)\, g_2(Re)$ then, the function $g_2(Re)$ can readily be evaluated from the thermally developed results and $g_1(Z/D)$ from new figures of $Nu(Z)$ versus (Z/D) being extensively pursued for various fluids and geometries.

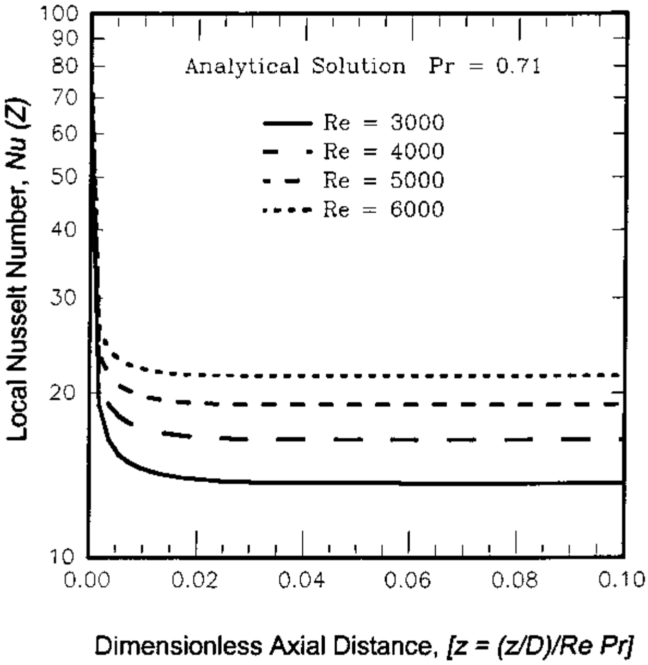

Figure 3.
Nusselt numbers in the thermal entry region of a circular duct for constant wall surface temperature and turbulent flow with $Pr = 0.71$ and $3,000 \leq Re \leq 6,000$

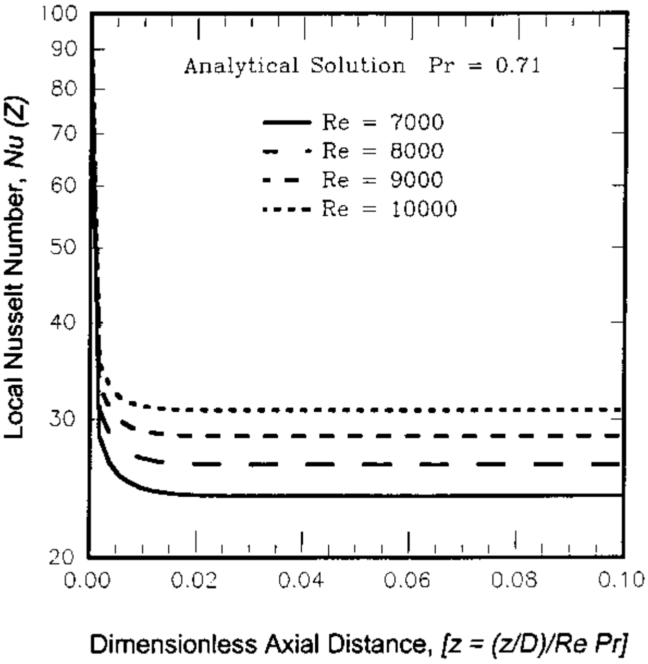

Figure 4.
Nusselt numbers in the thermal entry region of a circular duct for constant wall surface temperature and turbulent flow with $Pr = 0.71$ and $7,000 \leq Re \leq 10,000$

Conclusion

A novel theoretical approach, incorporating ideas associated with the generalized integral transform technique was developed for accurately predicating hybrid analytic solutions of fully developed steady forced convection in circular ducts with turbulent flow and constant wall surface temperature. Explicit analytical solutions for the thermal response of the fluid within the temperature field are reported systematically in terms of axial variations in bulk temperature and Nusselt number, covering a wide turbulent Reynolds number range. Further, it is shown that the thermally developed Nusselt numbers are equivalent in magnitude to the first eigenvalue squared.

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